# An Algorithmic and Software Framework for Applied Partial Differential Equations

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http://davis.lbl.gov/APDEC

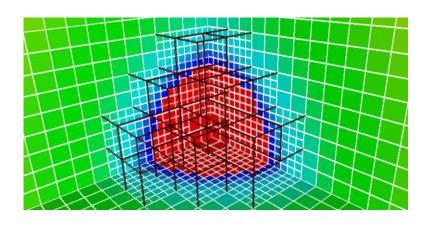
# An Algorithmic and Software Framework for Applied Partial Differential Equations (APDEC)

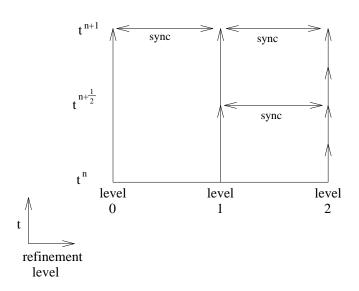
#### <u>Goals</u>

- Development of algorithms and software for PDEs based on locally structured grids. Technologies include adaptive mesh refinement (AMR), embedded boundary representation of complex geometries, and particle methods.
- Applications development: end-to-end development of new simulation capabilities for combustion, magnetic fusion, and accelerator design.
- Software development: factorized version of LBNL AMR tools to to maximize reuse across applications (Chombo). Leverage activities in other ISICs.

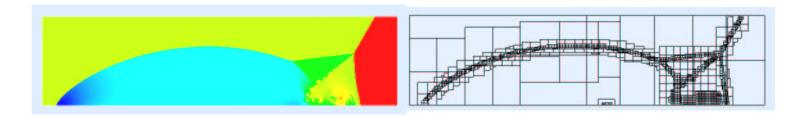
## Block-Structured Local Refinement (Berger and Oliger, 1984)

Goal: adjust the computational effort locally to maintain a uniform level of accuracy throughout the problem domain.



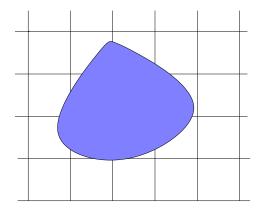


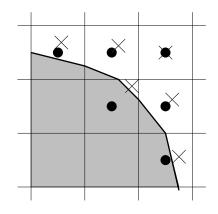
Refined regions are organized into rectangular patches. Refinement performed in time as well as in space.

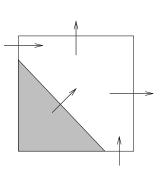


### **Cartesian Grid Representation of Irregular Boundaries**

Based on nodal-point representation or finite-volume representation.



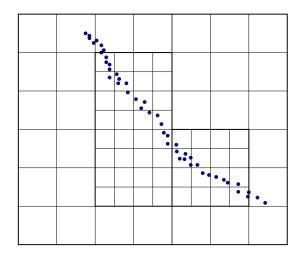




Advantages of underlying rectangular grid:

- Grid generation is tractable.
- Good discretization technology, e.g. well-understood consistency theory for finite differences. Geometric multigrid for solvers.
- Straightforward coupling to structured AMR.

#### Particles and AMR



## Algorithmic Issues:

- Transfers of particles across coarse/fine interface boundaries
- ullet Particle o grid and grid o particle transfers in the presence of refinement boundaries (modified stencils)
- preventing self-induced effects.

#### **Software Approach**

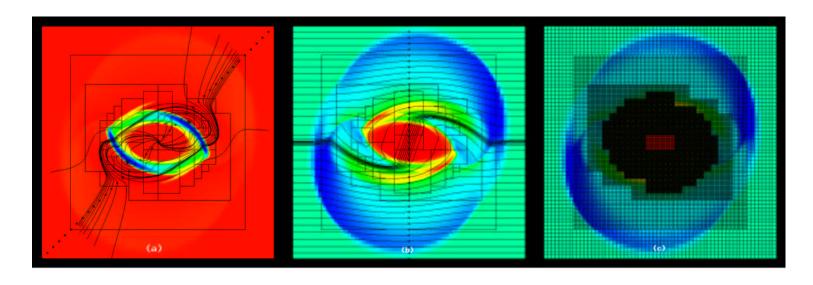
Requirement: to support a wide variety of applications that use block-structured AMR using a common software framework.

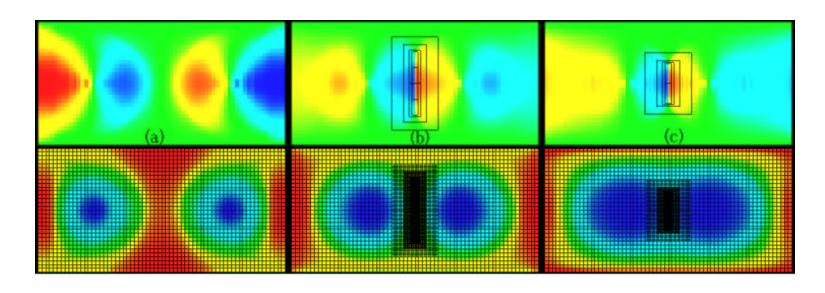
- Mixed-language model: C++ for higher-level data structures, Fortran for regular single-grid calculations.
- Reuseable components. Component design based on mapping of mathematical abstractions to classes.
- Build on public-domain standards: MPI, HDF5, VTK.
- Interoperability with other SciDAC ISIC tools: grid generation (TSTT), solvers (TOPS), performance analysis tools (PERC).

#### **Applications Progress**

- <u>Combustion</u>. Developed AMR codes in 2D and 3D for low Mach number combustion with complex chemistry, realistic diffusive transport. Obtained quantitative agreement of 2D laminar diffusion flame calculations with experimental  $NO_x$  measurements. Performed 3D time-dependent simulation capability of laboratory-scale turbulent methane flames (AMR + low Mach number leads to  $10^4$  reduction in compute time over standard compressible DNS).
- <u>Magnetic Fusion</u>. Developed explicit and semi-implicit AMR algorithms for resistive MHD in two and three dimensions. Performed high-resolution simulations of magnetic reconnection problem.
- <u>Accelerator Modeling</u>. Developed 3D AMR code for Poisson's equation with Shortley-Weller representations of irregular conducting boundaries. Developed preliminary implementation of AMR PIC code, and coupled to MaryLie / IMPACT beam dynamics code.

# AMR for Magnetohydrodynamics (R. Samtaney, PPPL)

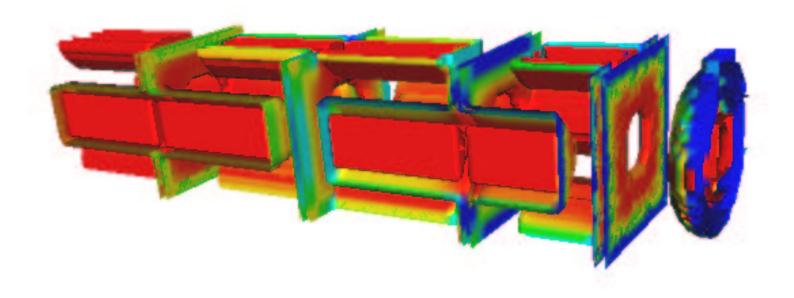




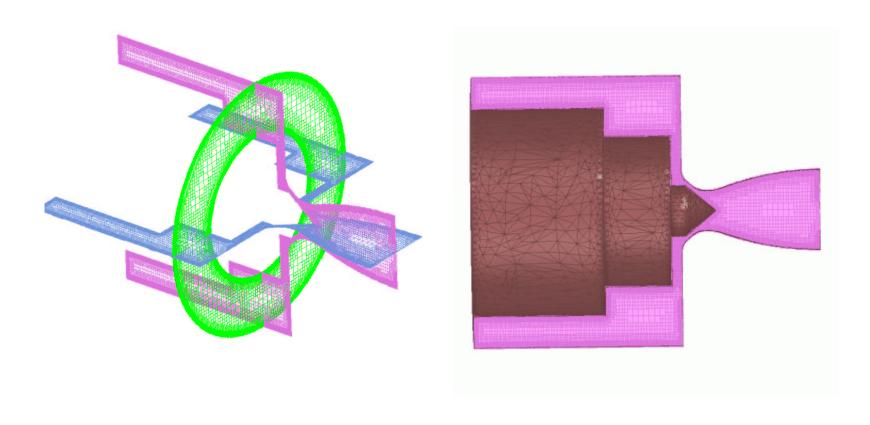
#### **Algorithmic and Software Progress**

- Improvements to Chombo package for AMR applications, including variable coefficient elliptic solvers, support for periodic boundary conditions, and new general-purpose hyperbolic drivers. Support for AMR particle methods.
- New stable and robust embedded boundary algorithms and solvers for PDEs in complex geometries.
- New tools for grid generation for embedded boundary calculations (joint with TSTT).
- Interoperability tools: framework-neutral AMR data alias; interface to HDF5 I/O, ChomboVis AMR visualization tools. Chombo interface to *hypre* structured-grid linear solver package from TOPS. Chombo interface to PAPI performance measurment tools from PERC.
- New semi-implicit spectral deferred corrections algorithms for complex time-dependent problems.

# AMR Shortley-Weller Solver for Poisson's Equation.



# Grid Generation for AAC Gas Jet Simulation Using Cart3D (M. Berger (NYU), A. Petersson (LLNL), C. Geddes (LBNL))



#### **Future Plans - Near Term**

- Combustion: comparison of 3D turbulent flame calculations to laboratory-scale flame experiments of Cheng, et. al.; coupling of spray models to AMR.
- Magnetic fusion: develop AMR code capability for pellet injection problem.
  Coupling of semi-implicit methods to models for reconnection.
- Accelerator modeling: development of AMR-PIC codes for beam dynamics based on alternative formulations. Fast evaluation version of James' method for infinite domain boundary conditions. Complete development of gas-jet simulation capability for laser-driven plasma-wakefield accelerators.
- Software: Complete development of embedded boundary AMR capability. Performance tuning of Chombo, e.g. introduction of level-based stencil operations, alternative grid generation, load-balancing strategies. Run-time grid generation for embedded boundaries. Demonstration project of componentization of Chombo elliptic solvers (joint with CCTTSS).

#### **Long-Term Plans / Opportunities**

- Development of AMR allspeed algorithms for low-Mach number / locally anelastic problems (combustion, supernova combustion, magnetic fusion).
- Combustion: extension to more complete nitrogen chemistry. Combustion in complex geometries.
- Extension of Cartesian grid methods to free boundaries, non-unit aspect ratios, other orthogonal coordinate systems (tracked flame fronts, plasma boundary in tokomak / stellarator geometries, gas jet boundary).
- Development of new algorithms / solvers for AMR grids: analysis-based scalable solvers for constant-coefficient problems, Newton-Krylov-Schwartz methods, conservative higher-order solvers (accelerator modeling, magnetic fusion).
- AMR for 5D / 6D phase space, hybrid fluid-kinetic modeling, Maxwell-Vlasov (accelerator modeling, magnetic fusion).